On the Magnetic Shielding of Large Spaces, and its Experimental Measurement.[*]

By ERNEST WILSON, M.Inst.C.E., M.I.E.E., Siemens Professor of Electrical Engineering, and J. W. NICHOLSON, M.A., D.Sc., Professor of Mathematics in the University of London.

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It is not a matter of difficulty to annul almost completely the field of the earth throughout a small space by a suitable choice of an iron shield. If, however, the region over which the field must be reduced is large, the problem becomes more difficult unless considerable masses of iron are used. For it is evident that the absolute dimensions of the shield are not important to the degree of shielding produced, that is to say, that a given shield magnified in all its parts in the same ratio continues to give the same degree of shielding. The object of the present investigation is to show that the shielding of large spaces can nevertheless be achieved by the employment of properly designed multiple or concentric shields, without the necessity for a prohibitive amount of iron. In connection with some recent work, moreover, the necessity has arisen for maintaining a magnetic field at least as small as $5 \,.\, 10^{-3}$ within a region of comparatively large dimensions. It is evident that the most suitable means is to reduce the field of the earth to this extent, but calculations soon show that the usual arrangements of soft iron shields involve a prohibitive amount of material. The necessity in fact arises for the determination of the most economical arrangement capable of giving a specified high shielding ratio. Since, in view of applications of the work, the actual magnitude of the field must be known, corrected for every variety of magnetic leakage which can invalidate the theory, some experimental device is also required for the measurement of such fields. There is apparently no record of the experimental determination directly of fields even of this order of magnitude, and since a method has been found which is capable of considerable accuracy and is in fact suitable for the measurement of fields only 10^{-4} times that of the earth, a description of this and of other methods with their practical limitations appears to be of value. It must be emphasised that the difficulties which are encountered, and the methods necessary to overcome them, are to a great extent peculiar and inevitable to work which must be carried on with large quantities of material.

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The investigation which originally led to the necessity for known fields of such a small order—it has since been supplemented by others which cannot at present be described—was the subject of earlier communications by one of us.*

In this investigation a shield was employed in which the ratio was of the order 0·07, calculated on the assumption that the average permeability of the material was about 100. For the purpose of further research, a very much smaller ratio was necessary, and other considerations required an internal diameter of the shield as large as 63 cm.

In the case of dimensions of this magnitude, the work hitherto published dealing with magnetic shielding is not applicable, for it has all related to the shielding of very confined spaces, where the present difficulties do not occur. Perhaps the most important work is that of du Bois,† but the subject is still insufficiently investigated in some essential respects. Du Bois has not determined the effect, for example, of a very small air space in a shell through which the earth's field can leak, and the experiments described in this paper indicate that the "leakage fields" which can enter the interior are of much greater importance than has been generally supposed. It is possible, as shown later, to separate these effects from the normal effect of the shielded field, and to study them individually by suitable arrangement of the air spaces, through which leakage takes place, with reference to the direction of the external field.

Most of the experimental work already done has, moreover, related to cylinders, which are obviously of no service in a case like the present, on account of the length involved. In the case of an iron core, for example, it is generally recognised that the length should be about 200 diameters, in order that the end effect should be negligible,‡ and a similar order of magnitude must apply to the present problem. Du Bois has used cylinders almost exclusively, and the fact that the lengths have apparently not been sufficiently near this limit—for example, the ratio of diameter to length was sometimes only 1 to 2·5—is probably one origin of the discrepancies which he found between the observed and calculated values of the shielding ratio, for the observed values are uniformly less than those calculated.§

In one paper du Bois and Wills have examined the effect of three concentric spherical small shields of different permeabilities|| and shown that

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* 'Roy. Soc. Proc.,' A, vol. 90, pp. 179 and 343 (1914) ; vol. 91, p. 104 (1914).

† 'The Electrician,' vol. 40, pp. 218, 316, 511, 652, 814 (1897–8).

‡ *Cf.* Ewing, 'Magnetic Induction in Iron and other Metals,' p. 233.

§ 'The Electrician,' vol. 40, p. 653.

|| H. du Bois and A. P. Wills, 'Ann. der Phys.' (ii), p. 78 (1900).

the results are in fair agreement with a general permeability equal to the geometric mean of the three when they range between 137 and 180. Such work cannot at once be extended to other cases, and in the present paper it has therefore been necessary to develop a general rapid mode of calculation of the shielding due to many shells of large radius before proceeding to an account of the experiments.

Laminated Shields.

If the earth's field is uniform and of intensity F outside a single spherical iron shell of permeability μ, and inner and outer radii a and b, the field inside is uniform and parallel to that of the earth, and of intensity*

$$9\mu\mathrm{F}/\{9\mu+2(\mu-1)^2(1-a^3/b^3)\},$$

where, as μ is in practical cases at least 100, $\mu-1$ may be replaced by μ with a maximum error of about 1 per cent. For iron of permeability 100 the shielding ratio which can be obtained with an unlimited quantity of material is only $2\mu/9$, or about 22, if a single shell is concerned.

The first important case in which lamination of the shell, or its division into two or more shells separated by air spaces, appears to have been used was at Greenwich, where Sir William Christie† protected the absolute magnetic instruments in this manner from the effect of a dynamo. This was done by covering the dynamo with a triple shield consisting of three arches with open sides, made of iron plates $\frac{1}{4}$ inch thick, separated by air spaces of breadth 3 inches. The three coaxial shields were placed over the dynamo with their open sides in the direction of the dynamo axis, and were all attached to one base plate. They could be closed entirely on one side by side-pieces of similar iron plate, but had a necessary partial opening on the other, on account of the dynamo shaft. The radius of the outer shield was 20 inches, and the axial length was also about 20 inches. With this arrangement, a shielding ratio of one-ninth could be obtained when the magnetometer needle was west of the dynamo, and one-seventh when it was north, the axis of the dynamo pointing north and south in both cases. The dynamo was of the two-pole type.

At that time the effect of lamination was imperfectly understood, more especially as regards the limitations to its utility for increasing the shielding ratio, and although these limitations did not apply to the actual shield devised by Sir William Christie, subsequent literature in connection with the shielding of instruments has frequently contained statements which appear to imply that further lamination is always an advantage. A paper by Sir Arthur Rücker, however, which has apparently been very much overlooked, was published so long ago as 1894,* and in this paper the necessary conditions which must be satisfied, in order that increased lamination of a shield may continue to be advantageous, were laid down precisely in several important cases.

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* Maxwell, 'Elect. and Mag.,' vol. 2, § 443, Edition 1873.

† Report of the Astronomer Royal, 1893 and 1894. 'The Electrician,' vol. 31, p. 446, August 25, 1893.

One of the most interesting general results of Rücker's investigation is that the reduction of the radius of the innermost surface always makes a considerable improvement in the degree of shielding. But it has apparently not been noticed that this and similar results can be grouped together into the formal statement that shielding is essentially a function of the curvature of the surfaces. Surfaces of high curvature, such as spheres of small radius, make very efficient shields, but it is difficult with a limited quantity of material to obtain great efficiency when the inner space must be so large that the average curvature of any enclosing surface is small. In such circumstances it is essential to deal with the problem of maximum shielding for a given amount of material and a given inner radius and number of shells. Rücker solved this problem in the case of two shells and showed that there is no "best arrangement" of two shells, unless the volume of the material exceeds about one-hundredth of the volume of the space enclosed, for a permeability of 500. If the value is only 100, we can show in the same way that the volume must exceed one-twentieth of the enclosed volume. When the volume of material used is smaller than this limit, lamination is actually a disadvantage, and, in general, for a permeability μ a shell should not be laminated if its thickness is smaller than the fraction $3/2\mu$ of its radius.

In order, therefore, to obtain good shielding in a space of radius 30 cm., every shell must have a thickness greater than about $\frac{1}{2}$ cm. if the permeability is 100, and unless the thickness were a reasonable multiple of this value the arrangement of shells would not be appreciably better than a single shell made up of the same amount of material. In the building up of a shield which shall be convenient in practice on a large scale this restriction is fundamental.

Rücker's investigation of the maximum only dealt with the case of two shells of equal permeability. It is evident that the more general problem of two shells of different permeabilities and the same number of variable radii as before cannot, for a given quantity of material, be treated in any definite way, unless the fixed quantities are specified numerically. This necessity for numerical values renders any investigation only applicable to one special case, and of little value in others, while the general formulæ cannot be dealt with in any practical manner. We have found it possible, however, to extend Rücker's investigation for equal permeability to the case of three shells. The analysis is somewhat long and cumbrous, and will therefore not be reproduced in this paper, as it follows obvious lines. It is perhaps sufficient to state that the results are essentially the same as for two shells. Lamination is again a disadvantage if the average thickness of the shells bears to the average radius a ratio so small as $3/2\mu$, but the range of unsuitable thickness is not very great, for a considerable advantage is secured when the ratio is only four times this value, provided that the air spaces introduced between the laminated shells also have a thickness at least as large as the mean thickness of the shells. These results are based on the supposition that μ is at least of the order 100, as in most cases of the initial permeability of iron.

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* 'Phil. Mag.' 5th Series, vol. 37, p. 95 (1894).

In the construction of shells of large radii, it is found very convenient to adopt equality of thickness throughout, and in this connection it is interesting to notice that the arrangement giving the best shielding effect is one in which the thicknesses of the individual shells are not very unequal. Moreover, in the neighbourhood of a maximum function of a set of variables, changes in the function are slow, and small alterations in the variables produce very small differences of the values from the maximum. This is noticeable in Rücker's calculations, and the more general investigation indicates that no important advantage is sacrificed if the shells are made equally thick, provided that their thickness is within appropriate limits. The appropriate lower limit, as stated above, is at least three or four times the "limiting thickness of lamination" $3/2\mu \times$ radius, with thicknesses of air space of the same order.

The exact multiple which must be used is determined by other circumstances, such as mobility of the apparatus, or total weight. In the experiments described in this paper, the multiple 4 is adopted.

Theory of a Series of Shells.

Although a formal expression as a determinant for the shielding due to n shells can be given, it is not very applicable to calculations. But when the permeability is large, a very close approximation can be calculated rapidly by a set of recurrence formulæ, of which an account is now given.

For a series of concentric spherical shells, surrounded by the field of the earth, giving a magnetic potential $\Omega = -\mathrm{F}r \cos \theta$, where F is the earth's force, the potential inside the hollow is $\Omega' = -\mathrm{F}'r \cos \theta$, where F/F' is the shielding ratio. Within the material of any one shell or within any air space, the potential is of the form $\Omega'' = (\mathrm{A}r+\mathrm{B}r^{-2})\cos\theta$, and for a large number of shells the recurrence formulæ must connect F and F′ through the sets of constants (A, B). The polar co-ordinates are taken at the common centre of the shells, with their initial line parallel to the undisturbed field of the earth. At each boundary of two media, the two usual conditions may be applied. The magnetic potential is continuous, implying that no work is done in carrying a small magnet for an infinitesimal distance across the surface. The normal magnetic induction is also continuous. Thus if Ω_1, Ω_2 are the potentials on either side of a boundary at $r = a$, the permeabilities being 1 and μ,

$$[\Omega_1 = \Omega_2, \qquad \partial\Omega_1/\partial r = \mu\partial\Omega_2/\partial r]_{r=a}.$$

Let a shell of permeability μ_1 have internal and external radii a_1 and b_1, and let a_2 be the internal radius of the next shell in the outward direction. We may connect the potentials at the distances a_1 and a_2 in the general case by a well-known method. If the potentials in the two air spaces are respectively $(\mathrm{A}r+\mathrm{B}r^{-2})$ cos θ (inner) and $(\mathrm{A}r+\mathrm{B}r^{-2})$ cos θ (outer), with $(\alpha r+\beta r^{-2})$ cos θ within the material, the preceding conditions give the relations

$$\mathrm{A}_1+\mathrm{B}_1/a_1^3 = \alpha+\beta/a_1^3, \qquad \mathrm{A}_1-2\mathrm{B}_1/a_1^3 = \mu_1(\alpha-2\beta/a_1^3)$$
$$\alpha+\beta/b_1^3 = \mathrm{A}_2+\mathrm{B}_2/b_1^3, \qquad \mathrm{A}_2-2\mathrm{B}_2/b_1^3 = \mu_1(\alpha-2\beta/b_1^3).$$

Let $\mathrm{P}_1 = \mathrm{A}_1+\mathrm{B}_1/a_1^3$, $\mathrm{Q} = \mathrm{A}_1-2\mathrm{B}_1/a_1^3$, then

$$\mathrm{A}_2+\mathrm{B}_2/b_1^3 = \tfrac{1}{3}\mathrm{P}_1(2+a_1^3/b_1^3)+\tfrac{1}{3}\mathrm{Q}_1(1-a_1^3/b_1^3)/\mu_1,$$

depending for a thin shell mainly on P_1 and not Q_1. Moreover,

$$\mathrm{A}_2-2\mathrm{B}_2/b_1^3 = \tfrac{2}{3}\mu_1\mathrm{P}_1(1-a_1^3/b_1^3)+\tfrac{1}{3}\mathrm{Q}_1(1+2a_1^3/b_1^3),$$

depending mainly on Q_1. For a large permeability and a thin shell we may write

$$1-a_1^3/b_1^3 = 3d/a_1,$$

where d is the thickness of the shell, and terms of relative order d/a_1 are neglected. In the experiments described subsequently, d/a_1 is about 1/15. If a second order is required,

$$1-a_1^3/b_1^3 = 3d/a_1-6d^2/a_1^2,$$

which may be denoted by $3\epsilon_1$, a purely geometrical constant of the shell. If this be substituted,

$$\mathrm{A}_2-2\mathrm{B}_2/b_1^3 = \mathrm{P}_1(1-\epsilon_1)+\mathrm{Q}_1\epsilon_1/\mu_1$$
$$\mathrm{A}_2-2\mathrm{B}_2/b_1^3 = 2\mu_1\mathrm{P}_1\epsilon_1+\mathrm{Q}_1(1-2\epsilon_1)$$

which are of a much simpler form. Accordingly, if a_2 is the inner radius of the next outer shell, calculation shows that

$$\begin{aligned}A_2+B_2/a_2^3 &= P_1\{1-\epsilon_1-\tfrac{1}{3}(1-\epsilon_1-2\mu_1\epsilon_1)(1-b_1^3/a_2^3)\}\\ &\quad +Q_1\{\epsilon_1/\mu_1-\tfrac{1}{3}(\epsilon_1/\mu_1-1+2\epsilon_1)(1-b_1^3/a_2^3)\}\\ A_2-2B_2/a_2^3 &= P_1\{2\mu_1\epsilon_1+\tfrac{2}{3}(1-\epsilon_1-2\mu_1\epsilon_1)(1-b_1^3/a_2^3)\}\\ &\quad +Q_1\{1-2\epsilon_1+\tfrac{2}{3}(\epsilon_1/\mu_1-1+2\epsilon_1)(1-b_1^3/a_2^3)\}.\end{aligned}$$

We may introduce a similar simplification for the air spaces. For if d_{12} is the thickness of air space between the first and second shells, and if

$$\epsilon_{12} = d_{12}/b_1-2d_{12}^2/b_1^2,$$

then

$$1-b_1^3/a_2^3 = 3\epsilon_{12}$$

and in general

$$1-b_n^2/a_{n+1}^2 = 3\epsilon_{n,\ n+1},$$

so that

$$\begin{aligned}P_2 = A_2+B_2/a_2^3 &= P_1\{(1-\epsilon_1)(1-\epsilon_{12})+2\mu_1\epsilon_1\epsilon_{12}\}\\ &\quad +Q_1\{\epsilon_1(1-\epsilon_{12})/\mu_1+\epsilon_{12}(1-2\epsilon_1)\}\\ Q_2 = A_2-2B_2/a_2^3 &= P_1\{2\mu_1\epsilon_1(1-2\epsilon_{12})+2\epsilon_{12}(1-\epsilon_1)\}\\ &\quad +Q_1\{(1-2\epsilon_1)(1-2\epsilon_{12})+2\epsilon_1\epsilon_{12}/\mu_1\}.\end{aligned}$$

These equations connect the constants in a convenient form at points just within two successive air spaces, and they are equivalent to recurrence formulæ. If they are developed further for thin shells of the order to which these experiments relate, $\epsilon_1 = \epsilon_{12} = \frac{1}{15}$ approximately, $\mu_1 = 100$, and within an error not exceeding about 2 per cent.,

$$\begin{aligned}P_2 &= P_1(1-\epsilon_1-\epsilon_{12}+2\mu_1\epsilon_1\epsilon_{12})+Q_1\epsilon_{12},\\ Q_2 &= P_1(2\epsilon_{12}+2\mu_1\epsilon_1-4\mu_1\epsilon_1\epsilon_{12})+Q_1(1-2\epsilon_1-2\epsilon_{12}).\end{aligned}$$

We may now write down the recurrence formulæ in a convenient form. Let u_n and v_n be the values of $A+B/a^3$ and $A-2B/a^3$ just within the air space bounding the inner surface of the nth shell, then

$$u_{n+1} = \alpha_n u_n+\beta_n v_n, \qquad v_{n+1} = \gamma_n u_n+\delta_n v_n,$$

where

$$\begin{aligned}\alpha_n &= 1-\epsilon_n-\epsilon_{n,\ n+1}+2\mu_n\epsilon_n\epsilon_{n,\ n+1}, & \beta_n &= \epsilon_{n,\ n+1},\\ \gamma_n &= 2\epsilon_{n,\ n+1}+2\mu_n\epsilon_n(1-2\epsilon_{n,\ n+1}), & \delta_n &= 1-2\epsilon_n-2\epsilon_{n,\ n+1}.\end{aligned}$$

These can be simplified if a lower order of accuracy is desired. If the field in the space internal to all the shells is F_1, then $B_1 = 0$ since it is finite at the centre, and

$$F_1 = A_1 = u_1 = v_1.$$

If the last outer shell is the nth, and the earth's field is F,

$$\mathrm{F} = \mathrm{A}_{n+1}$$

and $$u_{n+1} = \mathrm{F}+\mathrm{B}_{n+1}/a_{n+1}{}^3, \qquad v_{n+1} = \mathrm{F}-2\mathrm{B}_{n+1}/a_{n+1}{}^3,$$

whence, eliminating B_{n+1}, which represents the disturbance introduced externally by the presence of the shells,

$$\mathrm{F} = \tfrac{1}{3}(v_{n+1}+2u_{n+1}).$$

The shielding ratio is F/F_1 or $(v_{n+1}+2u_{n+1})/3u_1$.

In the calculation of the effect due to a gradually increasing number of shells, the work may be arranged very briefly as follows:—

If there is one shell, the ratio is $(v_2+2u_2)/3u_1$, where

$$u_2 = \alpha_1 u_1+\beta_1 v_1, \qquad v_2 = \gamma_1 u_1+\delta_1 v_1, \qquad u_1 = v_1.$$

The ratio is $(\gamma_1+\delta_1+2\alpha_1+2\beta_1)/3$ or, finally, $(3-4\epsilon_1+2\mu_1\epsilon_1)/3$, which is effectively Maxwell's formula. For two shells the ratio is $(v_3+2u_3)/3$, where

$$\begin{aligned} u_3 = \alpha_2 u_2+\beta_2 v_2 &= \alpha_2(\alpha_1 u_1+\beta_1 u_1)+\beta_2(\gamma_1 u_1+\delta_1 u_1) \\ &= (\alpha_1\alpha_2+\beta_1\beta_2+\gamma_1\beta_2+\delta_1\beta_1)\,u_1, \\ v_3 = \gamma_2 u_2+\delta_2 v_2 &= \gamma_2(\alpha_1+\beta_1)\,u_1+\delta_2(\gamma_1+\delta_1)\,u_1 \\ &= (\gamma_2\alpha_1+\gamma_2\beta_1+\delta_2\gamma_1+\delta_1\delta_2)\,u_1. \end{aligned}$$

The best illustration of the mode of calculation is a direct numerical one. Suppose, for example, we build up a series of shells, starting with a radius of 30 cm., and the thickness of the shells and air spaces is in each case 2 cm., the initial permeability of each shell being 100. Then, neglecting the second order, $\epsilon_1 = \frac{1}{15}$, $\epsilon_{12} = \frac{1}{16}$, $\epsilon_2 = \frac{1}{17}$, ..., and so on, while $\mu_1 = \mu_2 = \mu_3 = \ldots = 100$,

$$\alpha_1 = 1-\frac{1}{15}-\frac{1}{16}+\frac{200}{15\times 16}, \qquad \alpha_2 = 1-\frac{1}{17}-\frac{1}{18}+\frac{200}{17\times 18}, \quad \ldots$$

$$\beta_1 = \frac{1}{16}, \quad \beta_2 = \frac{1}{18}, \quad \ldots$$

$$\gamma_1 = \frac{200}{15}\left(1-\frac{2}{16}\right), \quad \gamma_2 = \frac{200}{17}\left(1-\frac{2}{18}\right), \quad \ldots$$

$$\delta_1 = 1-\frac{2}{15}-\frac{2}{16}, \quad \delta_2 = 1-\frac{2}{17}-\frac{2}{18}, \quad \ldots$$

and calculation shows that with values of this order of magnitude—which would always be the order in experiments on a large internal space such as those described—the differences between successive α's, β's, γ's, and δ's can be neglected, with a 2 or 3 per cent. error in the final results, at least for as many as eight shells or more, and thus effectively in the present numerical cases with 20 as the mean value of ϵ, we find

$$\alpha = \tfrac{3}{2}, \quad \beta = \tfrac{1}{20}, \quad \gamma = 9, \quad \delta = 1,$$

$$\begin{aligned}
u_2/u_1 &= 1{\cdot}5 & v_2/u_1 &= 10 \\
u_3/u_1 &= 2{\cdot}75 & v_3/u_1 &= 24 \\
u_4/u_1 &= 5{\cdot}5 & v_4/u_1 &= 49{\cdot}4 \\
u_5/u_1 &= 10{\cdot}65 & v_5/u_1 &= 98{\cdot}9 \\
u_6/u_1 &= 20{\cdot}92 & v_6/u_1 &= 195 \\
u_7/u_1 &= 41{\cdot}1 & v_7/u_1 &= 383 \\
u_8/u_1 &= 80{\cdot}8 & v_8/u_1 &= 753 \\
u_{n+1} &= \alpha u_n + \beta v_n, & v_{n+1} &= \gamma u_n + \delta v_n.
\end{aligned}$$

and the calculation continues rapidly by the recurrence formulæ. The shielding ratio for n shells being $(v_{n+1} + 2u_{n+1})/3u_1$ gives the following results with an external field of the earth equal to 0·5.

Number of shells.	Theoretical shielding ratio.	Earth's field as reduced inside.
1	4·33	0·115
2	9·88	0·0506
3	20·13	0·0248
4	40·1	0·0124
5	78·9	0·00634
6	155	0·00322
7	305	0·00164

The efficiency of the shield, whose arrangement is practically that of the maximum shielding, is approximately doubled by the addition of each shell after the first, even when the initial permeability is only 100. The average permeability in the later experiments is greater than 100, and increase of initial permeability beyond 100 rapidly increases the shielding ratio. For example, if $\mu = 150$, the first four values for the ratio with the present dimensions of the shells are found to be 6·03, 15·4, 34·8, 69·3, whose relative increase is more rapid than that of the preceding numbers.

For any other specified value of the permeability, such a Table as that shown above can be calculated in a very short time, so that the labour involved in the usual formulæ is now eliminated.

Construction of the Shield.

It was necessary, as stated already, that the shield whose efficiency is now to be tested should have an internal diameter of about 63 cm

After careful consideration, and in accordance with the discussion contained in preceding sections, the shield was designed to consist of a number of concentric spherical shells made of dynamo magnet steel of high permeability. There are four spherical shells, each of radial thickness 2 cm., and separated from one another by radial air spaces also equal to 2 cm. The inner diameter of the smallest shell is 63·5 cm. (25 inches), and the total weight of the shield is 1273 kgrm. (2806 lb.), made up as follows:—

	Actual weight.		Calculated weight.	
	kgrm.	lb.	kgrm.	lb.
1st shell	209	460	215	474
2nd "	305	672	270	596
3rd "	350	772	331	730
4th "	409	902	399	879

These are not quite in accordance with the theoretical weights corresponding to the dimensions of the original design. Each shell is in halves with plane butting surfaces of contact, which have not at present been machined. This has allowed an experimental determination, described later, of the effect of leakage. The air space between the two halves of each shell is sometimes as large as 3 or 4 mm. Extra metal is allowed to the extent of 1 cm. on each half in order that the surfaces can be so machined that each half penetrates the other, thus making each shell truly spherical. This dovetailing, when performed, will improve the shielding ratio by the removal of the main leakage effect.

The lower halves of the shells were maintained at the proper distance apart by the use of one gunmetal distance piece for each air space at the lowest point, where the weight had to be supported, and a series of wooden wedges around the circumference at the junctions. The upper halves rested freely upon the lower ones. A wooden frame with four curved arms supported the complete shield.

It was necessary to make use of an overhead crane for the handling of the shells, and a space was chosen in the dynamo room of the Siemens Laboratory at King's College, London, as far from the machines as possible. Tests with the inductor ultimately used in these experiments showed that the earth's magnetic field at the place chosen was practically undisturbed provided that the electrical machines were not magnetised. The maximum force due to the earth was found to be 0·473 C.G.S. units and its angle of inclination to the horizontal was about 67°. Screwed plugs of the same material as the shells were used to fill up the tapped holes used for the lifting eye-bolt—there was one hole of diameter 2 cm. in the centre of each half shell.

The Inductor.

For the measurement of small magnetic forces, du Bois* has described a permeameter which was used in his determination of shielding ratio. This instrument makes use of a suspended magnet controlled by a fixed one after the manner of an ordinary reflecting galvanometer, and records deflections by the motion of a spot of light on a divided scale. An instrument of this type appears to possess serious disadvantages, for the presence of a permanent magnet inside the shield must polarise the material to some extent and thereby alter the distribution of magnetic intensity in the interior. When the shield is capable of reducing the earth's field in its interior to a very small amount, the effect of the magnet becomes relatively more important. The use of such an instrument, however small the magnet may be, is therefore restricted to cases in which the shielding ratio is small. It was mainly for this reason, but also through considerations of sensibility, that another method was chosen for the measurement of the internal force. The sensibility is not, in fact, very great, for the lowest force measured by du Bois in this manner is only 1/70 of that of the earth's field. Some experiments which we performed with steel magnets of various sizes suspended by quartz fibres indicated that the analogous method of oscillations is not readily capable of producing better results.

A second method of measuring the internal fields on which experiments have been made involves the use of a rotary air-core armature, supplied with a commutator and brushes for direct-current measurement. The armature actually employed in these experiments had an effective area of 130 sq. cm., and was wound with 100 sections, each consisting of 50 turns. It was of the 2-pole type, and the commutator had 100 parts. When the armature was rotated at 3600 revolutions per minute a maximum voltage of 0·37 could be obtained in the earth's field, and the method is obviously a very sensitive one. But it was abandoned on account of the mechanical difficulties of moving the brushes into known positions when inside the shield, and driving the armature at high speed.

The final method adopted consists of the use of an inductor rotated by hand, in conjunction with a ballistic galvanometer. Before the weight of copper required was definitely decided upon, preliminary calculations were made, and it was found that with a given mean diameter of the inductor and a given instrumental resistance the rate of increase in sensibility per unit weight of copper began to decrease at about 13·6 kgrm. (30 lb.) as the weight of copper was steadily increased. With a weight of 13·6 kgrm. it would have been difficult to start the coil from rest, turn it through 180°, and bring it to rest again in a time small compared with the periodic time of the galvanometer needle, and it was therefore decided to continuously rotate the coil and to employ a sector and brush which makes contact during the half-revolution. With a speed of rotation of about one revolution per half second it was possible to close a key in the circuit by hand, and thus impress upon the instrument the electromotive force developed during one half-revolution. The brush is supported by a disc, which is provided with a divided circle, and is capable of being turned through known angles. In this way the direction as well as the magnitude of the force was determined.

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* 'The Electrician,' vol. 40, p. 815.

The two insulated conductors connecting the inductor coil with the sliding contacts were threaded through a brass tube of diameter 1·3 cm. This tube served as the axis of the inductor, and passed through a circular hole in each shell. The hole is compounded of two semi-circular holes, one cut in each half-shell at the junction. It is important to notice that in this manner the axis of rotation of the inductor is always in the plane of the junctions of the individual shells.

Adopting a scale having ½ mm. divisions at a distance of 1340 mm. from the mirror, the constants of the instrument employed in preliminary tests are given in Table I. Throughout the experiments, double deflections were observed.

Table I.

Instrument.	Periodic time in seconds.	Ampères per scale division (steady deflection).	Instrumental resistance at 10° C. in ohms.	Coulombs per scale division.
Open type, 4-coil......	7	$0·125 \times 10^{-6}$	4	$0·139 \times 10^{-6}$
,, ,, 2-coil......	22	$0·336 \times 10^{-7}$	24·4	$0·118 \times 10^{-6}$
D'Arsonval	8	$0·75 \times 10^{-9}$	467	$0·955 \times 10^{-9}$

Table II gives the calculated size of wire and number of turns in order that in each case the deflection may be a maximum. The last column gives the scale divisions obtainable when observing the full value of the earth's field. Since a throw of 10 scale divisions can be measured with some accuracy, a field which is only one ten-thousandth of that of the earth can be determined with the d'Arsonval instrument, whose sensibility is evidently very much greater than that of the others. It is worthy of mention that although even greater sensibility can be obtained with this type of instrument, it is only possible when certain compensating disadvantages are introduced. The most important, perhaps, is the "creeping of the zero," due to a permanent set in the suspension fibre. In fact, if difficulties of this kind are to be avoided, it would seem that the sensibility obtained with the present instrument is near the practical maximum.

In the experiments with a temporary winding on the wooden former ultimately used for the inductor, it was only when the coil was placed with its plane in the neighbourhood of parallelism with the direction of the earth's field at the time of making and breaking contact that a periodic time of 22 sec. gave the more reliable result, otherwise the short-period instruments were equally suitable. But since the main object of these experiments was to obtain maximum sensibility, the d'Arsonval galvanometer was chosen.

The particulars of the coil finally used are as follows:—Resistance at 11° C., 400 ohms; mean diameter, 57·7 cm.; radial depth, 3·2 cm.; width, 6·75 cm.; total number of turns, 3227; number of layers, 36; diameter of copper wire, 0·558 mm.; mean area × number of turns $= 8{\cdot}43 \times 10^6$ sq. cm. The wire was insulated by a double covering of cotton.

Table II.

Instrument.	Diameter of copper wire.	Weight of copper.		Resistance of coil and galvanometer.	Number of turns.	Deflection in scale divisions.
	mm.	kgrm.	lb.			
Open type, 4-coil	1·829	159	35	8·56	758	9,040
,, ,, 2-coil	1·219	148	32·6	45·9	804	4,210
D'Arsonval	0·5588	117	25·9	867·0	3227	104,000

Testing of the Shells as Received from the Foundry.

(*a*) Experiments were made with the junctions of the shells in a horizontal plane, and the axis of rotation of the inductor at right angles to the magnetic meridian. The force which is measured in the shells is, of course, an average value over the volume occupied by the inductor. If there were no leakage this would be the constant value of the theoretical uniform field inside. Lack of uniformity may cause a small dependence of the registered value on the size of the inductor. In fig. 1, Curve 1 shows the relation between this average magnetic intensity inside the first or smallest sphere and the position of the brushes. This position is taken as the zero or standard of reference for the measurement of angles. The plane of the coil is vertical when the inclination is 0°, at the moment at which contact is made or broken during the one half-revolution which causes the deflection.

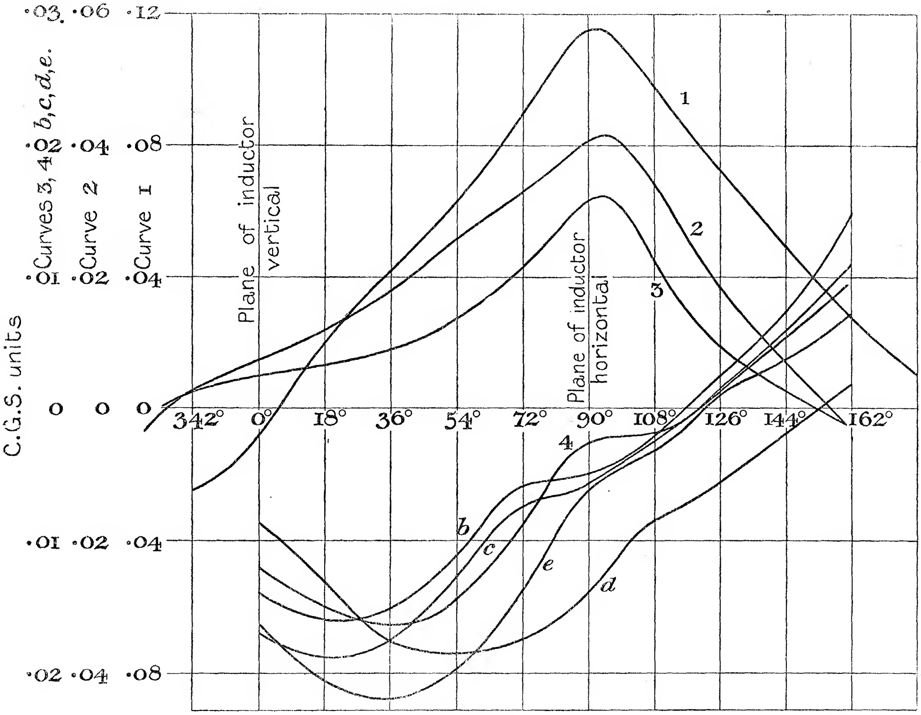

Fig. 1.

At an inclination of 90° the coil is horizontal. It will be noticed that the curve is not a pure sine function of angle, and that the maximum intensity occurs when the coil is horizontal, and not when it is at right angles to the direction of the earth's field, which is at about 67° to the horizontal. The point at which the curve crosses the horizontal axis is in advance of 337°, at which angle the electromotive force due to the earth's field outside is zero. Fig. 1, Curve 2 was obtained when the first shell was placed concentrically inside the second, Curve 3 when the third was then placed over the second, and so on to Curve 4, when the four shells were complete. The maximum ordinates of Curves 2 and 3 are still vertical, but they cross the horizontal axis more and more nearly at the angle 337° = 270°+67°. Curve 4 is almost completely reversed in sign, with respect to the others. With a view to elucidating the origin of this curious reversal of sign, the following variations were made in the experiments, in each of which the four shells were used simultaneously.

(*b*) The junctions of the four concentric shells were maintained in one plane, which was at right angles to the direction of the earth's field, and at 67° to the horizontal, and the axis of rotation of the inductor was at right angles to the meridian. Under these circumstances Curve *b* was obtained.

(*c*) The junctions of the four concentric shells were kept in the same plane, which was inclined at 58° to the vertical, and was, together with the axis of the inductor, at right angles to the meridian. In this manner Curve *c* was obtained.

(*d*) The junctions of the first and third shells were inclined at 76°, and of the second and fourth at 58°, to the vertical. All these planes were, together with the axis of the inductor, at right angles to the meridian. The result is shown in Curve *d*.

(*e*) All the junctions were in a horizontal plane, but the axis of the inductor was in a horizontal plane in the meridian. Curve *e* was obtained in this way.

The above experiments tend to show that the position of the junction planes in relation to the direction of the earth's field does not account for the reversal of sign of the internal field in the case of the four complete shells. For more especially in (*d*), the field which leaks through the first junction plane is incident on the solid mass of the next shell except at two single points only, and therefore should not appreciably leak into the next shell. The same considerations come into play in the interior shells. It seems fairly evident, therefore, that the effect is due to permanent magnetisation of the outer shell, and in a later section this supposition is verified.

The approximately horizontal portions shown on Curves 4, *b*, *c*, *d*, *e*, in the neighbourhood of 90°, are due to the tendency of the earth's field to preserve the right sign against the opposing forces. It will be noticed that in all the experiments with the planes of junction identical, the relative positions of the shells are the same on account of the axis of the inductor. Any change in the position of this axis would have involved further necessary perforation of the shells, and consequent increase in the leakage. This important variable was for this reason not tested.

Experiments with Magnetising Coils.

The foregoing experiments demonstrated that, although the presence of the air spaces between the butting surfaces of the shells was in part responsible for the effects observed, the iron was also partially polarised owing to its having experienced the full force of the earth's field before being placed in position and possibly during the cooling immediately after it was cast at the foundry. In order to remove this effect of polarisation, each shell was wound with a magnetising coil around the diameter of the concentric shells which coincided with the direction of the earth's field. Each coil fulfilled the condition that the number of ampère-turns per unit length of the diameter normal to the plane of the windings was constant.

The field inside the winding was therefore constant in strength, and had the value $8\pi nx/3$; where the total number of turns is $2na$, n being the number per unit length, a the radius, and x the current.* Copper wire of diameter 0·08 cm., insulated with cotton, was used for these windings, and it was supported by being wound into a series of hooks, fixed to tape stretched around three meridian planes of each shell. The numbers of turns wound on the respective shells, starting with the smallest, were 34, 38, 42, 46, the value of n being $\frac{1}{2}$. The experiments may be classified as follows:—

(α) Experiments were first made with the fourth or largest shell alone. The plane of the butting surfaces was placed at right angles to the direction of the earth's field, and the axis of the inductor was at right angles to the magnetic meridian. In fig. 2, Curve A was obtained before passing current through the magnetising coil of 46 turns, and it is notable that its maximum ordinate occurs more nearly at 67° than in fig. 1, Curve 1. Currents of gradually increasing values were then passed through the magnetising coil in such a direction as to oppose the earth's force, and for each current a curve was obtained. The curves are distinguished in fig. 2 by the current

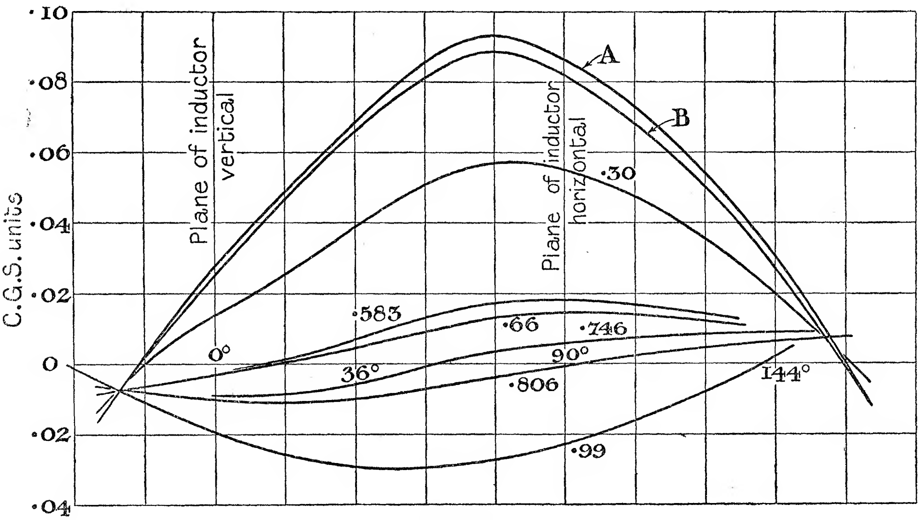

Fig. 2.

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* See Maxwell's 'Elect. and Mag.,' vol. 2, § 675, Edition 1873.

in ampères which was flowing during the observations, ranging from 0·30 to 0·99. It will be noticed that the reversal of force within does not occur by passing through the value zero through the whole volume of the interior, but by a gradual shifting of phase unaccompanied by any very important change in amplitude. It will also be noticed that the wave-form of Curve A is more nearly a sine curve than was the case in fig. 1; this is believed to be due to the fact that the diameter of the inductor is considerably smaller than the inside diameter of this particular shell, and the leakage near the inner surface therefore does not contribute to the electromotive force generated by the rotation of the inductor. We have suggested already that if the internal field is not strictly uniform, the size of the inductor may play a part. It can, in fact, be seen by inspection of the curves that the same influence is at work as in the earlier experiments. When the opposing current is 0·746 ampère, the component in the direction of the earth's field is zero, and there is a field associated with the air space between the opposing surfaces which has a maximum value of 0·009 C.G.S. unit at right angles to the direction of the earth's field. For the curve for this value of the current cuts the axis very close to the maximum.

When the current had reached the value 0·99 ampère, and the final curve had been obtained, the current was reduced gradually by a series of steps to zero, and at each step reversed 20 times. The number of steps was 40 in each case of demagnetisation, and the interval of time between each reversal was 4 seconds. It was found that this time was necessary on account of the delay in reversal caused by eddy currents. The final current was 0·0004 ampère before the circuit was broken. Curve B was obtained after breaking, and it shows that the shielding is slightly improved by the operation. The Maxwell formula requires a current of 1·2 ampères to annul totally a force of about 0·5, which can be taken to be the maximum due to the earth; but in this theoretical case the surface is more closely wound than in the experiments, and moreover, unit permeability is assumed throughout the space.

(β) The four complete shells with their magnetising coils in position having their junctions in the same plane, and this plane being at right angles to the direction of the earth's field, and the axis of the inductor at right angles to the meridian, were next subjected to various tests. A large number of experiments were made with the magnetising coils in order to remove the effects dealt with in connection with fig. 1, Curve 4. The following operations have been made to remove the residual effect, and are given as a typical example.

Referring to fig. 3, Curve 1 will serve as a starting point. Curve 2 was obtained after making and breaking a current of 6 ampères in the fourth or outermost magnetising coil, the direction being such as to oppose the earth's force. A current of 3·5 ampères was then passed through the first magnetising coil on the innermost shell in such a direction as to oppose the earth's field, and was then broken. Curve 3 was then obtained, and shows that the inner shell, owing to residual magnetism, has reversed its internal field. The depression in the neighbourhood of 67° is due to the effect of the earth's field superposed on that of the shell, which is now polarised in a new manner. A current of 0·746, which had been found to be sufficient to neutralise the earth's field in connection with fig. 2, was now passed through the outermost coil so as to oppose the earth's field. It was maintained during such time as the inner three shells (whose coils were in series) were demagnetised from 3 ampères to 0·0004 ampère. With this current of 0·746 ampère still flowing in the outermost coil Curve 4 was taken. The current in the outermost coil was then broken, and Curve 5 was obtained. At this stage there are now no currents flowing in any of the magnetising coils, but the outermost shell still requires demagnetisation from the value corresponding to 0·746 ampère, which was the last applied current. It will be noticed that this curve has a peak near 67°, due to the earth's field, and this peak must represent the true shielding effect. The great depression at 0° is due to the leakage field, the effect of which was shown

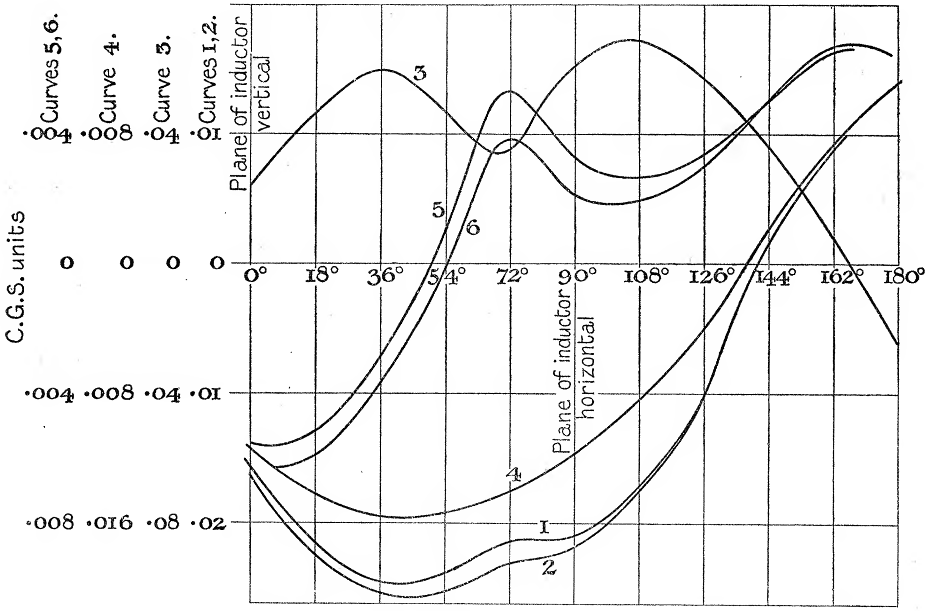

Fig. 3.

clearly in connection with fig. 2, and the peak beyond 160° is due to the same cause.

Finally, the outermost shell was demagnetised from 0·746 ampère down to 0·0004 ampère, and Curve 6 exhibits the result. This curve gives a force of 0·0038 C.G.S. unit in the direction of the earth's field, and the field at right angles associated with the air space has a maximum of 0·0063 C.G.S. unit. These experiments completely confirm the original supposition that the remarkable behaviour of the outermost shell, as shown in the curves depicted in fig. 1, was due to polarisation.

The leakage field of 0·0063 C.G.S. unit is practically 1/83 of the outside field of the earth, and by this final set of experiments it has been possible to isolate it and to subject it to precise measurements. A rough estimate would suggest that, for one shell, the leakage field should bear to the external field a ratio of the same order as the ratio of area of leakage space to total area of shell. The average outer surface of a shell is $4\pi(36\frac{1}{2})^2$ cm.^{2} and the circumference of a diametral section is $2\pi(36\frac{1}{2})$ cm. The thickness of leakage space varied between 0 and 4 mm., with an average 0·2 cm. The corresponding ratio is $4\pi(36\frac{1}{2})^2/2\pi(36\frac{1}{2})(0{\cdot}2) = 340$.

Thus the leakage actually found for four shells is four times as great as might have been expected for one by this rough argument, which is evidently therefore inapplicable, and leakage fields must therefore be guarded against in such experiments more thoroughly than has usually been thought necessary.

An inductor which practically fills the internal space is of course necessary in the estimation of the true average value of the leakage, which is by no means uniform throughout the space.

Using the results obtained for one, two, and three shells without demagnetisation, the intensities in the shielded fields are as follows :—

Number of shells.	Intensity.	Number of shells.	Intensity.
0	0·473	3	0·016
1	0·116	4	0·0038
2	0·041	—	—

with, in the final case, a leakage field of 0·0063 in the perpendicular direction in C.G.S. units.

General Discussion.

A comparison with the illustrative case worked out in a previous section from the formulæ indicates that the average permeability of the four shells must be high. Before the final shield was built up and subjected to demagnetisation, preliminary experiments were made on the smallest and largest shells singly, and they were found to give shielding ratios 4·086 and 5·175 respectively, their inner radii being 31·5 and 42·5 cm. The first value when compared with the theoretical Table indicates a lower permeability than 100. The actual values calculated from the formula, or from Maxwell's formula, are respectively $\mu_1 = 82{\cdot}4$ and $\mu_2 = 148{\cdot}9$, the second being much larger. The difference cannot be explained by the relatively small divergences of the weights from the theoretical values, and it is in fact an illustration of the difficulty of casting shells of different sizes but of equal permeability. Another practical difficulty is the prevention of a permanent polarisation during the original cooling of the shells, which in this case had not been annealed. The largest shell was obviously, from its experimental behaviour, at first magnetised to an appreciable extent, and the probable existence, though on a smaller scale, of a similar effect in the others renders these values, originally obtained for the permeability, liable to error in defect.

From the permeability of the first shell we can, by the use of the observed shielding ratios of the various combinations, deduce the permeabilities of the second and third. The calculation follows obvious lines from the preceding formulæ, and it is sufficient to state the final results, which are $\mu_2 = 163$, $\mu_3 = 148$, subject to an error owing to the uncertainty of μ_1, which is probably larger. Knowing the permeability of the fourth shell, we can then deduce the theoretical shielding ratio for the four shells in combination. The result is 0·0055 for the internal force in the combination, and this is an upper limit owing to the nature of the data. It is in sufficiently good agreement with the experimental value 0·0038 after demagnetisation, which must improve the shielding. The shielding thus obtained is as perfect as it should be from the theoretical point of view, and the results indicate that when the leakage has been eliminated by the dovetailing of the junctions, the maximum possible shielding effect—in accordance with the theory outlined earlier—for this weight and quality of iron will have been attained. This maximum effect is of the order necessary for the developments contemplated, and it is demonstrated that the method devised for the measurement of these smaller fields in large spaces is satisfactory. The way is now open for the final determination of the behaviour of iron under fields which are practically infinitesimal.

Summary.

1. The magnetic shielding of a large space is a problem wholly different in practice from that of a small space, and in view of important applications

the efficiency to which such shielding can be raised is a matter of importance. Considerations of mobility of the apparatus and weight of iron required necessitate the solution of the problem of maximum shielding for a given weight of iron and more than two shells, together with an examination of the limitations of utility of lamination. These problems are discussed in the paper.

2. A field of order so low as $3 \cdot 10^{-3}$ has been obtained in a space of radius 30 cm., by the use of 1273 kgrm. of high permeability dynamo magnet steel, and an accurate method designed for the measurement of fields of lower order.

3. The leakage through air spaces in a magnetic shield has been studied, and found to be more important than is usually supposed. Although polarisation of the shells still exists to a small extent, it is not of sufficient magnitude to affect any of the conclusions which have been reached. The leakage field can, in fact, be completely isolated from other fields.

4. It is now possible to examine the behaviour of iron under practically no magnetic force.

The Period of a Spherical Resonator with a Circular Aperture.

By F. Puryer White, St. John's College, Cambridge.

(Communicated by Lord Rayleigh, O.M., F.R.S. Received June 23, 1916.)

In a paper on "The Theory of the Helmholtz Resonator,"* Lord Rayleigh has carried the determination of the wave-length of the fundamental aërial vibration in a spherical vessel with a small circular perforation to a second approximation, obtaining the result

$$\lambda = \pi\sqrt{(2\mathrm{S}/a)} \cdot \left\{1 - \frac{9}{10}\frac{a}{2\pi c} + \ldots\right\}, \tag{1}$$

where c is the radius of the sphere, a the small radius of the aperture, and S the volume of the sphere. To obtain this value he assumes a form for the normal velocity U over the aperture and adjusts it so as to lead to agreement, to a corresponding approximation, in the values of the velocity potential ψ derived therefrom over the aperture inside and outside, so as to provide for the condition of continuity of pressure at the opening.

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* 'Roy. Soc. Proc.,' A, vol. 92, p. 265 (1916).